DYNAMICS OF POPULATION AND RESOURCES

VERY SHORT SUMMARY OF THE BOOK BY MARIO LUDOVICO

PUBLISHED WITH THE TITLE "DINAMICA DI POPOLAZIONE E RISORSE"

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1. Introduction and synthesis

Since a very long time, there is a human need for satisfactory explanations about the natural laws that govern the appearance, growth and (sometimes) disappearance of biological populations, particularly concerning human populations. At variance with possible expectations, there is no easy answer to such a licit curiosity.

The answer given in the 18th century by Malthus[1], with his exponential growth law for human population, has been accepted as a reasonable one up to a relatively recent time, notwithstanding the meaningful correction introduced by Verhulst[2] a few decades later.

Nowadays, the analysis of population dynamics has become a field of highly sophisticated theoretical exercises, though most of the visible results leave much to desire. Demographers have still to tackle serious problems, when requested to deliver reliable projections.

The contents of the book regarded by this summary is also one of these exercises, thought the degree of sophistication has been minimised, with a view to providing more manageable analytical instruments. In this connection, it is worth pointing out that the model of ecosystem presented here has no pretence to scientific work. The aim is only to introduce a relatively simple and clear logical framework in analysing the relationship between populations and the resources used for survival and development purposes.

The finding of this analysis leads to the following two major conclusions:

(i) no stable equilibrium between any population and its environment is in general to be expected:

(ii) whatever population, whose number of components does not decline with time, tends to extend constantly the mass of its resources along with the bounds of its ecosystem.

2. The basic logical framework of the model

With respect to any given population, the model represents the relevant ecosystem as a set of three major sectors that interact with each other as well as with themselves.

The ecosystem's components are grouped according to the following sectors:

(1) *Population* (i.e., the study population)

(2) *Resources* proper to the population

(3) *Environment*, i.e., all the remaining part of the ecosystem that doesn't belong either to the population or to its resources.

The interactions between these three sectors, and inside each of them, consist of specific flows of biological and non-biological materials *and* energy in various forms.

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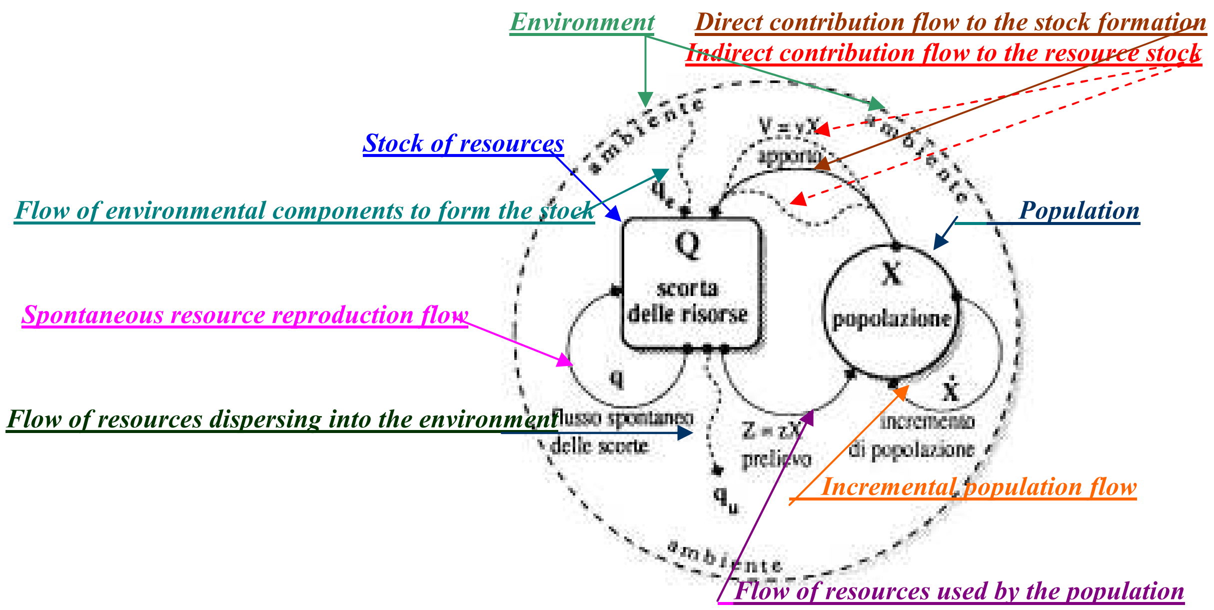
[1] Thomas Robert Malthus (1766-1834), *Essay on the Principle of Populations as it Affects the Future Improvement of Society,* London 1798

[2] Pierre François Verhulst (1804-1846), Belgian Mathematicians, formulated the so-called *logistic function*, to express the concept that population growth is constrained by environmental resources, first of all by the limited amount of physical space (territory) pertaining to each particular population. It is interesting to remark that the work by Verhulst remained actually ignored for nearly a century.

The *Environment* is the fundamental source of raw materials that go to form the specific *Resources*. The *Resources* “feed” the *Population*, which, in its turn, directly and/or indirectly, returns part of discarded materials to the *Environment*, while, as it is for example with human populations, does also “feed” its own stock of resources through the transformation of materials taken from its *Resources*. The schematic representation of this ecosystem is given by the figure below.

A GRAPHIC SCHEME OF THE MODELED ECOSYSTEM RELEVANT TO ANY GIVEN BIO-POPULATION



In this figure, “flow” means “amount of materials, energies, resources, population, etc. that transfers from one sector of the ecosystem to another (or to the same sector) in a given time unit”.

2.1 The basic hypotheses

The model logic framework results from the processing of three hypotheses:

(I) Variation dQ/dt in the resource flow of the resources Q absorbed by the population mass X, can be either a linear function of population increment dX, or a linear function of the acceleration in the incremental population flow dX/dt: i.e.,

either $d(dQ/dt) = \lambda\, dX$

or $d(dQ/dt) = \mu\, d(dX/dt)$

in which λ and μ are two coefficients of proportionality, and t indicates time;

(II) The flow of spontaneous (autonomous) resource regeneration $q = dq^*/dt$ varies with time according to a periodic trend, i.e.:

$$q = dq^*/dt = q_1\cos^2 ut - q_2\sin^2(ut + \varphi/2).$$

In this equation, q^* represents the mass of those materials and energies that belong to the *Resources* sector Q, but not used by *Population* X, for they remain absorbed by the resource sector itself for self-reproduction ends; q_1, q_2, u and φ are positive constants;

(III) The flow Z of resources absorbed by the population is a function of the population mass X, as follows:

$$Z = (s + \sigma y)X,$$

in which y is the population reproduction rate at instant t, s is the average resource consumption per population unit when $y = 0$, and σ is the increment (positive or negative) in the average resource consumption per population unit when $y \neq 0$.

In parallel to the function above (and through mechanisms proper to the environment), the direct or indirect contribution V provided by the population to the regeneration of its resources is expressed by

$$V = (r + \rho y)X ,$$

in which r is the average rate of restitution per population unit when $y = 0$, and ρ is the increment (positive or negative) in the average restitution rate, per population unit, if $y \neq 0$.

The model equations

The hypotheses formulated above lead to the interesting conclusion that the dynamics of the ecosystem may be summarised and described by a simple linear differential equation of the following form

$$\alpha dX/dt + bX + k\cos(wt+\psi) + g = 0 , \quad (*)$$

in which α, b, k, w, ψ, and g are real constants.

From the linear differential equation above, and through a series of other relationships and definitions, one obtains different functions that express the possible evolution of both population X and relevant resources Q, according to the different values that can be established for the equation constants.

In particular, for population X the following evolution equations can be written:

$$X = A + B\,e^{-ct} + C\sin(wt+2\psi),$$
if $\alpha \neq 0$ and $b \neq 0$

$$X' = A' + B't + C'\sin(wt + \psi),$$
if $\alpha \neq 0$ and $b = 0$

$$X'' = A'' + C''\cos(wt + \psi),$$
if $\alpha = 0$ and $b \neq 0$.

In the preceding equations, A, A', A'', B, B', B'', C, C', C'', and ψ are integration constants that depend on the border conditions. Real constant c in the exponential is not nil.

In correspondence with these population dynamics equations, the following equations can be written for the dynamics of the relevant resources Q, i.e., respectively:

$$Q = Pt + Ne^{-ct} + N\sin(wt + 2\psi) + R,$$
if $\alpha \neq 0$ and $b \neq 0$

$$Q' = P't^2 + M't + N'\sin(wt + \psi) + R' ,$$
if $\alpha \neq 0$ and $b = 0$

$$Q'' = P''t + N''\cos(wt + \psi) + R'',$$
if $\alpha = 0$ and $b \neq 0$.

Also in these equations, all the quantities that differ from independent variable t are integration constants, with $c \neq 0$ in the exponential.

It is not possible to justify here the conclusions mentioned in points *(i)* and *(ii)* of the introduction.

The subject is instead addressed and discussed in the book text.

In case of availability of the necessary statistical data, the equations showed above can be used for non-linear regression analyses and projections.