TOWARDS A COMPLEX-SYSTEMS ECONOMICS

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Abstract

Political economics, in developed societies, had its origin as a mere philosophical thought focused on social behavior; it was initially indistinguishable from theories concerning social organisation and ethics. In Europe, physiocratic doctrines, Liberalism, Socialism – for example – were substantially philosophical theories aimed at replacing religion in justifying political regimes or initiatives, social reforms and even revolutions, as a consequence of the transformation in the way of thinking that was brought about by the development of positive sciences. A number of European philosophers became convinced that both human history and societies could be investigated scientifically, like any subject of positive science, such as, for instance, chemistry, physics, medicine, etc. Such a belief is still hard to die, despite undeniable failures and tragic events undergone by billion people because of it. Social systems – as conditioned by local traditions, culture, resources and individual aspirations – are "complex systems", to mean phenomena that escape the human capability of final, objective and complete understanding**:** the perception of them is intrinsically affected by a predominant amount of uncertainty. A humble awareness joined to a cautious use of such a constraint might improve the effectiveness of our methods for understanding, with no ontological ambition and to the common benefit, what we can observe of complex self-organised systems. A simulation theory is here proposed to try an unbiased description of mental processes concerning the study of complex socio-economic systems.

1. Introduction

Political economics has its historical origin as a philosophy focused on the behavior of human communities in developed societies, and was initially undistinguishable from theories concerning social organisation. Physiocratic doctrine, Liberalism, Socialism and other currents of sociological thought were substantially theoretical systems aimed at justifying political reforms and even revolutions, as a consequence of the transformation in the way of thinking that was brought about by Enlightenment in 18th Century, in conjunction with an extraordinary progress of positive sciences and development of technological innovation. From Locke, Smith, Fourier, Owen, to Marx, Comte, Mills and several others, a number of European philosophers became convinced that both human history and societies could be investigated scientifically, like any subject of positive science, such as, for instance, chemistry, physics, medicine, etc. Positivism and Neo-Positivism followed and dominated the philosophical debate for about half a century. The exceptional development of mathematics and statistics between the first half of the 18th Century and the first half the 20th Century have corroborated the socio-economists' conviction that sociology, economics and even politics could be the subjects of scientific, objective and unbiased analysis. Such a belief is still hard to die, despite undeniable failures and social and economic disasters undergone by billion people in the world, as occurred because of the pretence to apply "scientific" criteria to the organisation and life of political communities. One example for all**:** Marxian analysis is still largely thought of, by its supporters, as an example of "scientific" socio-economic analysis, against which *more modern* (either Keynesian or neo-Keynesian or monetarist, for example) theories of political economics seem still inadequate. I doubt it is possible to apprehend, from either Marxian, post-Marxian or other contemporary schools of economics, of any movement of thought inclined to admit that theories of political economics do still persist in a philosophical sphere that has no connection with positive science.

The power of positive science consists in its capability to predict events, basically through calculation, with a high level of accuracy and to control physical phenomena to such a point to allow anyone, thanks to the technological output of applied sciences, to objectively benefit from the scientific achievements upon individuals' demand. In this connection, it is important to remark that positive sciences are used to deal with phenomena which can be represented, though schematically, with quite a limited – though experimentally sufficient – number of known parameters and variables**;** whereas the study of socio-economic systems, each with its own historical, cultural, sub-cultural, political and geographical identity, have proved impossible to be *summarized* and scientifically represented by means of a limited number of parameters and variables such as private and/or public investment capital, labor offer and employment rate, per-capita income, growth rate, monetary circulation, propensity to consume, marginal utility, population growth, demand elasticity, inflation rate versus unemployment, production functions, innovation impact, and a few other additional variables that complete the list conventionally addressed by theories and models of political economics.

Furthermore, most macro-economic indicators and parameters that are considered as significant is assessing aspects of developed and democratic countries, have quite often very low or no significance in underdeveloped countries, where development is more often an issue than a process, and political regimes have little or nothing to do with democracy and human freedom.

Thus, however high the theoretical persuasion power of economics, the facts are there, every day and especially today, to show that it is a field of studies unable both to make useful objective predictions on a secure theoretical basis and to control socio-economic processes at will.

What might appear paradoxical is that in recent decades economists have showed off theories and models built up by use of highly sophisticated mathematics**;** but such abilities are not *per se* sufficient to turn the mathematical reasoning based on arbitrary assumptions and abstract simplifications into scientific stuff.

Nevertheless, it seems also impossible to renounce any attempt to understand the behavior of human communities. Economics provides models for would-be effective interpretation of our common socio-economic behavior, because it is indispensable to understand something of what is going on, in a view to undertake any political as well as individual initiative.

Statistical economics first, and econometrics later, have been and still are reasonable ways to respond to our demand for understanding.

Certainly, the analysis of relationships between quantifiable events, i.e., the use and processing of statistical and other observational data regarding events and effects of social life form a more rational approach to the issue. Statistical analyses of economic processes are at least the best way to corroborate or – to the contrary – question and confute theoretical models proposed by economists. However, I do not know how many economists are aware of that statistics does inevitably introduce a crucial component of uncertainty in the analysis and interpretation of the study subject.

What I mean is that human communities are very complex *systems of interactions* between local institution, between institutions and individuals, and between individuals, all in turn heavily conditioned by local history and individual stories, culture, tradition, beliefs and expectations, climate, geography and much more, so that statistical analysis and data processing and econometric models, wherever possible, cannot avoid to omit too many significant *as well as fickle* aspects of each particular community.

Pre-selected theoretical options, cultural formation and prejudices do always bias the way in which we tend to represent the reality we observe, even when our basic purpose is to avoid any reference to general philosophical criteria. Yet, the situation would be even worse if theories and models concerning human societies could identify, assess and incorporate all imaginable variables and parameters**:** an insoluble problem would arise, because of the impossibility of establishing in principle the correct way to put them in relation with each other. Attempts of a similar kind, which regard another group of *complex systems*, characterise nowadays models of ecosystems, aimed at predicting the destiny of our planet's climate. The result consists in a remarkable confusion (strongly and obviously denied by the model builders), according to which almost *anything* – and the relevant opposite – may be *predicted*.

Unfortunately, as a regional planner involved in the preparation of development programs for several countries in different continents, I have had more than one occasion to be amazed by the dullness of "experts" from schools of economics of Western Countries**:** "experts" used to go – firmly relying on their pre-made conceptual tool-kits – to advise governments, especially of the Third World, affected by critical socio-economic conditions.

Shall we remind ourselves, for example, of the economic disasters that followed "advices" (they were actually imposed conditions) given by IMF or World Bank "expert economists" to Mexico, Argentina, Ethiopia, Russia, countries of South East Asia and others in recent years?

No eco in those experts' minds of the severe public self-accusation made by Milton Friedman[1] in 1972:

"In our capacity of economists we have caused major damages to the whole society and to our profession too, in promising more than that we can give. We have encouraged politicians to make odd promises and to

[1] M. FRIEDMAN, *Have monetary policies failed?,* in The American Review, 1972, LXII

infuse groundless hopes, since the results [*of the policies suggested*], though sometimes acceptable, remain far from the economists' Promised Land".

2. The spirit of econometrics

Econometrics tries to respond to the need for a closer approach to the study subject through any available quantitative measurement method and data processing instrument. Econometrical analysis focuses on the "quantities" identified both as statistical data and measurement results. Basic instruments of econometrics are multivariate analysis and correlations, with the relevant search for possible links between hypothesized causes and respective effects. The limits of these particular instruments are in the intrinsic "passivity" of the analyses, especially concerning macro-systems**:** any possible diagnosis or projection is – on the one hand – based on a drastically subjective data selection, generally with reference to particular theories or models of economics; and – on the other hand – econometric instruments have a substantially weak predictive power, because of the unjustifiable assumption that things tend to indefinitely follow the path they have gone so far.

Certain theoretical choices – such as, for instance, the criteria for data selection or the assumption of trend continuity – give almost immediate evidence to the observational and computational omissions.

Now, experience teaches that "complex systems" are just those systems on which the classifiable information is too little, fuzzy and intrinsically unstable for allowing analysts to make *deterministic* predictions**:** which is equal to say that "stable laws of behavior", capable of connecting precise causes to definite effects, are for such systems impossible to identify. *Evolution* of complex systems is something much more complicated than a *tendency* to behave.

After the monumental work of statistical economics carried out by Simon Kuznets, the first important attempt to take econometrics to the level of a comprehensive and unbiased methodology was made by Wassili Leontiev through his macro-analysis of the inter-sector relationships of a national economic system. The method sticks to the *measurement* of the interaction flows between sectors of a given economic system, with no other "prejudice" than the criterion adopted for identifying the various inter-related sectors.

To note: such a criterion alone is already sufficient to undermine the full objectivity of the analysis though.

The only hypothesis (and the crucial technical limit) of the method is that the *input* of each sector is made directly proportional to the respective *output*. In principle, concerning the production system, the hypothesis is hardly questionable**:** everybody would agree, for instance, that the amounts of coal, mineral materials, labor, energy, capital money, transport loads, etc. are directly proportional to the amount of steel produced**;** and so on for other sectors. The practical problems in applying the method arise when each of the identified sectors does not consist of a single type of production plant, but – because of an inevitable need for simplification – gathers the output of several different activities, which are *akin* but not *identical* to each other. So that the inter-sector transactions cannot be measured in homogeneous product units (e.g., in tons, or cubic meters, etc.) but only as transaction flows expressed in monetary units. Additional practical difficulties intervene when the analysis aims at long term predictions, which cannot necessarily account for the immanent *disturbing* role of technological innovation and unforeseeable changes in the price/cost of some inputs or in governmental policies.

Notwithstanding the inherent practical difficulties, Leontiev's conceptual approach to the economic macro-analysis is revolutionary, in that it does not break down the study system into selected conventional economic categories (labor, capital, investment, marginal utility, demand, offer, market equilibrium etc.)**:** instead, the analysis limits itself to identify and account for transactions between different *activities*, intrinsically and objectively measurable irrespective of their nature and of any cause or end that determine them. The methodological scheme, in other words, may be applied to any society and economic system, provided that the basic assumption is verified, i.e., that a certain degree of inter-dependency between the different activities exists. In itself, Leontiev's inter-sector analysis has no reference to any particular school of economic thought. Beyond all possible criticism, it is an important attempt to free macro-economics from philosophical speculation, with a view to keeping the observation of a complex system within a *least-biased* conceptual reference frame.

As known, after the original scheme proposed by Leontiev, the method has undergone a significant number of improvements and adjustments, and the *input-output* inter-sector analysis has been adopted by many governments for managing national accounts. It is a fact that the method, despite the approximations associated with the hypothesis of linear dependency between the system's activities, provides analysts with a useful *calculation* instrument to get credible short term indications about the expected impact on the whole system caused by possible alterations in the activity of one or more of its sectors. No other model can provide the analysts with a credible *objective* indication of what impact, for instance, on clothing industry could be expected from an increased investment in automobile industry, or what impact on fishery production could be connected to a decrease in the family savings.

Actually, the method constitutes the first *usable* instrument of *complex systems analysis*. The observation and measurement of interactions between human activities, along with the identification of the functional nature of the relationships, accounts for *all* that which motivates and determines the behavior of the members of a self-organised human society, including the *chaotic* set of individual intentions, prejudices, errors and superstitions. *All this* is completely, as well as indistinguishably, expressed by the intensity of the measurable transactions.

3. A further step

The methodological jump made by Leontiev in addressing macro-economic issues is an encouraging suggestion to go further along the conceptual path he has indicated.

Leontiev's inter-sector analysis, as already remarked, is affected by one ill-working *functional* hypothesis, the one regarding the "technical coefficients" of direct proportionality between inputs and respective outputs. The analytical need for aggregations of various *akin* different activities makes the direct proportionality not only questionable, but systematically unstable with time, mainly – but not only – because of frequent alterations in the price set of the production factors along with unforeseen productivity changes in some of the activities considered. The method would be quite adequate, especially as for short run projections, if the "technical coefficients" would be constant quantities. Unfortunately, experience has widely shown that it is not so. This fact has actually implied a complicated and endless work of formal adjustments of the method together with a continuous activity of updating of the set of values forming the matrix of technical coefficients.

In years Seventy and Eighty of the past century, my professional work of regional analyst and planner has led me to re-consider Leontiev's methodological approach to complex systems from a more general point of view. I thought it was appropriate to exploit the fundamental importance of each interaction flow, this viewed as the conveyor of *all* the information inherent in the specific relationship it represents and expresses.

Upon the only assumption that not all the interaction flows between well identified components of a complex system are random flows, i.e., assuming in general that part, if not all, of those interaction flows are caused and motivated by specific ends (which I dub "intents"), I found it is possible to configure a mental image of the study system in a *quasi-neutral* way. Such a "neutrality" is affected only by the limits of the language we use both to describe what we observe and to process the findings of our observations. However, every language is the inherited basic instrument generated by the culture we live; it is used not only to represent but also *to understand* the reality we experience.

In other words, the perception of any object or set of objects occurs *both* through a physical contact (i.e., through senses and instruments) *and* through languages that can represent and describe the object perceived. It's just through the language that one can determine the modes of concentration and distribution of his attention.

The *linguistic institutions*, which pre-exist individuals and generations, not only determine a shared communication medium between different observers, but also - to a very large extent - *a shared way in which the world is perceived*. It's a physiologic *datum* that transcends individual mental attitudes and induces many to believe *naturally* that each of the terms and concepts, which belong to the languages used, are *objectively* corresponding to *things*, these being therefore perceived as objects that *pre-exist per se*.

The above premise intends to introduce the awareness that the identification, the definition and the description of whatever "system" is substantially a linguistic operation of a subjective nature.[2]

Complex human systems emerge and evolve because of "local constraints" that prevent possible interactions between members of a human set from being all random and meaningless. This statement implies that our mental activity inclines to use a concept like "degree of order" in observing "anything" we are able to classify as "system". Any system is such, in our view, to the extent to which we do not perceive it simply as a chaotic set.

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[2] M. LUDOVICO, *Syntropy. Definition and Use*, in online magazine www.syntropy.it, December 2008, No.1, p. 158. Other link: www.mario-ludovico.com/pdf/syntropy.pdf

The description of the behavior of a system depends principally on the criteria adopted for identifying its components.

The identification of the components does also determine the representation of the system's configuration, i.e., the seized distribution of the interactions within the system.

The salient property of any system is that all its components are *active*. The interactions that regard any system are both those that occur between different components of the system and those of each component with itself (*self-interaction)*.

In this connection, it's worth remarking that also the "external universe", though theoretically not clearly identifiable in its own features, shall nevertheless be considered as existing and included in the set of the system's components, whatever the system. Also such "external component" generates (and confines in itself) a *self-interaction,* which consists in the amount of activity supposedly due to its relationships with the other well identified components of the study system.[3]

Under the condition that the interactions between the system's components are all identifiable and measurable, the description of the system's behavior becomes simpler than one could expect.

In analyzing any system, the observer is used to focus his attention only on those interactions that are significant for the study's purposes.

Then, in describing a particular behavior of the system, it is obviously supposed that the interaction flows, as observed in a given time unit, are methodically quantified by use of a measurement system that renders all interactions homogeneous quantities, in order to make them comparable to each other.

A subsequent important consideration is that all the individual interaction flows, if these are *not stable* by hypothesis, can be converted into *interaction probabilities*, which also enable the analyst to exploit some mathematical properties of a probability distribution.

Besides any possible discussion on the *meaning* of this kind of probability distribution, the *percent values* expressed by such probabilities (i.e., the ratio of each flow to the total amount of flows in the system) are significant enough to justify the relative use in the analysis. In particular, interactions expressed in the form of "probabilities" are useful to the purpose of expressing the intrinsic amount of informational *uncertainty* associated with the system's states.

Actually, the subjective assessment of a probability distribution depends only on the information with which the analyst is provided through the measurement of the interaction flows.

If we now apply the set of concepts expressed above to the representation of a regional or national economic system we do actually adopt the Leontiev's approach to the macro-analysis of the system. The intervening differences are in the supposed nature of the observed activities, and in the identification and description of the *objective constraints* that characterise the economic system as a self-organised system. Basically, economic inputs and outputs are both viewed as *transactions,* i.e., as *action flows* moved by the *intent* to achieve quantifiable *benefits*, whatever the nature of these.

The new approach is no more deterministic, but *probabilistic***:** sector inputs and outputs are supposed to be *possible* events, each occurring with a variable probability**.** The relevant probabilistic nature consists in that such flows are *not* considered as *stable events*, but only as *possible events* whose *temporary* intensity is detected through surveys conducted during states of *precarious equilibrium* of the system's configuration.

The theoretical paradigm outlined in the next paragraphs may be considered as applied to a *closed* economic system. The economic system becomes "closed" by subdividing the "final sector" of Leontiev's scheme into two sectors: (1) the "families sector", viewed both as "labor provider sector" and as "consumer sector"; (2) the whole set of foreign countries viewed both as source of the system's imported products and destination of exported products.

In this way, the transaction matrix of the economic system is a square matrix of NxN transactions, N being the number of the different specific sectors of the system identified, there including the "external sector", which is the origin and destination of the main system's imported and exported products, respectively.

[3] For the determination of the *self-interaction* of the system's "external component" see M. LUDOVICO, *L'evoluzione sintropica dei sistemi urbani" (Syntropy and Evolution of Urban Systems),* Published only in Italian language by Bulzoni, Roma 1988 (2nd ed. 1991), pp. 208-223.

4. **Basic theoretical features of a new method**

The theoretical framework of the new method recalled here is as follows.

This *simulation theory* regards only a *mental representation* of any possible complex self-organised system. It is *not* the representation of any *real system*.

Of course, any economic system may in particular be thought of as a complex self-organised system.

The theoretical description of such a *mental representation* of complex system rests on a few basic assumptions.

The first assumption is that the *interaction flows* between elements of the considered system are *physically measurable*.

The second assumption is that a quantifiable *expected intent* is associated with each interaction.

The third assumption is that *all that is known* about interaction flows is expressed

(i) by the physical measurement of the flows,

(ii) by the formal definition of the relevant "intents", and

(iii) by those relationships, between any flow and other flows, which can be identified and expressed formally.

On the basis of these three major assumptions, the theory derives the fundamental equation that puts every interaction (or transaction flow) into a mutual univocal relationship with the "intent" that motivates the same flow. The equation is:

[1] $$T_{jk} = \frac{hO_j D_k e^{m_{jk}}}{T};$$ (valid for any *j* and *k*)

where:

T_{jk} is the measurement of the ***flow*, i.e., the transaction per time unit,** which is originated by system component *j* and bound for component *k* ;

m_{jk} is the measurement of the *expected intent* associated with the same flow. As shown ahead, these intents are completely determined by the given distribution of the system's transactions.

O_j is the total amount of transaction flows generated by *j* in the time unit considered (in an economic system it represents the *total output* of sector *j* per conventional time unit);

D_k is the total amount of the system's transaction flows that are bound for *k* during the given time unit (in an economic system it represents the *overall demand* of sector *k* for commodities and services per conventional time unit);

T is the overall amount of transactions generated by the system in the same time unit (i.e., the system's total output);

h is a coefficient that depends on the system's equilibrium state, if any.

The following formal definitions concern some of the quantities introduced above:

[2] $$O_j = \Sigma_k T_{jk};$$ (valid for any *j*)

[3] $$D_k = \Sigma_j T_{jk};$$ (valid for any *k*)

[4] $$T = \Sigma_j \Sigma_k T_{jk} = \Sigma_j O_j = \Sigma_k D_k .$$

To shift from flow absolute measurements to *flow probabilities* it is sufficient to divide Equations **[1]** by total flow *T*, to obtain:

[5] $$P_{jk} = \frac{T_{jk}}{T} = hP_i Q_k e^{m_{jk}};$$ (valid for any *j* and *k*)

which expresses the probability of a transaction flow from *j* to *k,* once defined

[5a] $$P_j = O_j / T \;;\quad Q_k = D_k / T\;;$$ (valid for any *j* and *k*).

P_j is the probability for component *j* to generate a unit flow (of output) during the fixed time unit, while Q_k is the probability for component *k* to be the destination of any unit flow generated (demanded) by the system during the same time unit.

It is also immediately seen, because of relations **[4]**, **[5]** and **[5a]**, that

[6] $$\Sigma_j \Sigma_k P_{jk} = 1 .$$

This equivalence indicates that the set of the flow probabilities associated with the system may be considered as a *probability distribution*.

Equations **[1]** are obtained by maximizing the probabilistic uncertainty *E* associated with discrete probability distribution **[6]**, under all the quantifiable constraints that affect this probability distribution. The constraints are expressed by equations **[2], [3], [4],** and by the following equation

[4a] $$\Sigma_j \Sigma_k u_{jk} T_{jk} = U$$

in which u_{jk} is *the mean effect expected* in association with transaction T_{jk} . Concerning an economic system, quantities u_{jk} may be viewed as the *mean economic benefit expected* in association with one unit of transaction T_{jk} . U is the expected overall benefit per time unit associated with the system's activity.

At this point in the discussion, it is of a fundamental importance to draw attention to the fact that *definitions* **[2], [3], [4], [6]** and *assumption* **[4a]** constitute *all* that the analyst is supposed to know for sure about the study system. All other possible information is too fuzzy and uncertain to be clearly formulated and steadily associated with the complex system's activity, so that *no additional assumption* can *in general* be clearly formulated and proposed as *systematically true.* Therefore, apart from the four definitions and the hypothesis **[4a]** mentioned above, the analyst's uncertainty is maximum as to the indefinite myriad of contingencies upon which the system's activity forms and develops.

In other words, the uncertainty in describing the system would be maximum (i.e., the analyst's information about the system would be nil) if there were no constraint to limit the randomness of the interaction distribution between the system's components, as otherwise would be perceived by the analyst.

"Intent" m_{jk} is the relative *expected effect* u_{jk} multiplied by constant λ , which is a Lagrange multiplier determined through the constrained maximisation of the probabilistic uncertainty associated with the probability distribution defined by equation **[6].**

Lagrangian multiplier λ is a positive constant quantity that depends on the measurement system adopted by the analyst. In this connection, however, it is worth observing that in most practical applications the numerical determination of λ is not necessary, "intent" $m_{jk} = \lambda\ u_{jk}$ being already in itself quite a significant indicator.

As already indicated, intent m_{jk} is a measurement of the mean unit "economic purpose" associated with the respective flow T_{jk} . For the theory, the value of m_{jk} may vary between $-\infty$ and $+\infty$**.**

The complete set of values m_{jk} (i.e., the NxN matrix $\{ m_{jk} \}$) determines the *system's structure.* It is the system's network of *expectations*.

The concept of *probabilistic uncertainty*[4] is substantially the concept of *information entropy* defined by Shannon and Weaver in 1949, and is expressed here by

[7] $$E = - \Sigma_j \Sigma_k P_{jk} \mathbf{Ln} P_{jk}$$

"**Ln**" is the symbol for natural logarithm.

Function **[7]** (*uncertainty* or *entropy)* is then the quantity to be maximized (by *Lagrange multipliers method*) under the constraints – as indicated above – which can be written to express *all that is known* about the considered interaction flows.

Probabilistic uncertainty, or entropy, is a positive quantity which is *always* associated with any probability distribution and can be expressed *only* through function **[7]**.

Given the measurement of all the interaction flows, an important implication of Equations **[1]** is that the quantification of the *expected intent* associated with each flow – as previously announced – is also univocally determined. In fact, from **[1]** one obtains:

[8] $$m_{jk} = \mathbf{Ln} T_{jk} - \mathbf{Ln}(O_j D_k / T) - \mathbf{Ln}\ h\ ; \quad \text{(for any } j \text{ and } k)$$

and it can be proved that

[9] $$-\mathbf{Ln} h = 2\mathbf{Ln}(N/T) + (1/T)\ \Sigma\ (O_i\ \mathbf{Ln} O_i + D_i\ \mathbf{Ln} D_i).$$ [5]

N is the number of components that form the system. Coefficient h has no physical dimension and pertains to any "intrinsically unstable equilibrium state" in which the system can be described by Equations **[5]** (whereas *transition phases* - which are inherent in *transformation cycles,* are described by subsequent Equations **[21]** to **[26]**).

Parameter h, whose value may vary between 0 and 1, can be thought of as the probability for the system to change its state.

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4 C. SHANNON & W. WEAVER, *The Mathematical Theory of Communication,* University of Illinois Press, Urbana, 1949.
In its original formula, uncertainty – or entropy – includes a constant coefficient that depends on the logarithm base**:** it has here been assumed as equal to 1 .

5 The whole mathematical discussion concerning the theory summarised here, with the relevant theorems and proofs, is in my book, *L'evoluzione sintropica..",* op. cit.

From the theoretical point of view, it is important to remark that Equations **[8]** and **[9]** imply that the interactions between the components of ***any*** system may in general – at least to some degree – be viewed as *intentional*, considering that $m_{jk} = 0$ means *no intent*. If all the elements of matrix { m_{jk} } are nil, then $h = 1$, necessarily.

Instead if $h = 1$, it is easily proved that

$$T_{jk} = (O_j D_k / T) = T / N^2 , \quad \text{(valid for any } j \text{ and } k)$$

Actually, $h = 1$ characterises the *extremely unstable equilibrium state* of maximum disorder, as it is expressed by maximum uncertainty $E_{Max} = 2\mathbf{Ln}N$.

In this connection, h is interpreted as the probability for the system to change its state. For any "system", the state of "total disorder" is by definition meaningless and therefore impossible.

4.1 - Other basic definitions

The maximum value of uncertainty (disorder) is associated with probability distribution **[6]** if all probabilities P_{jk} are equal to each other. In this case, the flow probability between any pair of components is a constant expressed by $P = 1/N^2$. That is why, because of Equation **[7]**, the maximum value of uncertainty is expressed by

[10] $$E_M = 2\mathbf{Ln}N = H .$$

However, as remarked above, E_M cannot affect any system, since "systems" may form only if $E \neq H$.

The theory considers uncertainty E as a measurement of perceived disorganization (*disorder*) in the system and, therefore, H expresses a theoretical *limit state* of the system, about which nothing remarkable can be said except that it is extremely unlikely or – better – substantially impossible. Such a limit state is also referred to as the system's *entropy potential.*

Relation **[10]** indicates that quantity H depends only on the number of the different components that form the system. This fact draws attention to the importance of the criteria used for identifying-describing the system.

In any perceived state of the system, the difference S, between entropy potential H and uncertainty E, is taken as a measurement of the system's degree of *order* or *organization* in that particular state, and is defined as the system's "*syntropy*". Therefore, *syntropy* is

[11] $$S = H - E .$$

Syntropy[6] provides a means for measuring the degree of organization (*order*) in the system, and any change in the system's syntropy gives an indication on the overall "improvement" or "worsening" undergone by the system upon simulated (or recorded) alterations in its hypothesized (or observed) states, under the conventional assumption that *order* is better than *disorder.*

In this connection, a significant indication from the simulation theory[7] is the relation between the system's syntropy S and the overall benefit U associated with the system's activity. The relation is given by

[11a] $$S = \lambda \frac{U}{T} = \lambda u$$

T being the sum of all interaction flows**.** λ is the Lagrangian multiplier whereby "intents" are defined, and u is the expected mean benefit per transaction unit. The relation draws attention to the very close relationship between the concept of "degree of order/organisation in the system" and the concept of "expected benefits" associated with the transaction flows that characterise the system.

Going back to the probability distributions, consider now that also the probability distributions expressed by relations **[5a]** imply a probabilistic uncertainty associated with each of them. The set of quantities **[2]** and **[3]**, (i.e., *outputs* and *demands* in an economic system) or, alternatively, the set of the two respective discrete probability distributions { P_j } and { Q_k }**,** is here called "base of the system", and the sum of the relevant "uncertainties", as defined by

[12] $$E^* = -\sum (P_i \mathbf{Ln} P_i + Q_i \mathbf{Ln} Q_i) ,$$

is the "base entropy" of the system. In general, E^* differs from E.

If, for any j, is $P_j = Q_j = 1/N$, then the base entropy becomes $E_M^* = H$, and therefore no system exists for the observer.

[6] The term "syntropy" was first introduced by matematician Luigi Fantappie' in 1945, to mean that "*quid*" which brings organisation in any physical (especially biological) process, in an apparent contrast with the Third Law of thermo-dynamics. In the same year, for analogous purposes, Max Plank suggested the term "negentropy", but the relevant concept differs from that inherent in "syntropy".

[7] M. LUDOVICO, *L'evoluzione sintropica ...*, op.cit. pp.225-226. A concise summary in English is available at: http://www.mario-ludovico.com/pdf/syntropy.pdf

Thus, analogously to definition **[11]** above, it is also possible to define the "base syntropy" of the system as

$$S^* = H - E^*.$$

Base syntropy S^* is in general different from syntropy S, though the following relationship is constantly true**:**

[13] $$E + S = E^* + S^* = H\,.$$

The following equivalence is also true of any unstable *equilibrium state* of the system:

[14] $$S^* = -\,\mathbf{Ln}\,h\,,$$

and justifies the name of "stability" for base syntropy S^*. If $h = 1$, "stability" becomes nil, which occurs – as already seen – if the system's entropy $E = H = 2\mathbf{Ln}N$ is maximum.

It is worth observing that Equation **[14]** leads, through Equation **[8]**, to express every flow intent m_{jk} also as a function of the system's stability S^*.

(Note: This conclusion conflicts with the properties conceptually associated with the maximum entropy of thermodynamics. According to classic thermodynamics, any isolated system – and the Universe itself – tends to a final equilibrium state that establishes at the maximum entropy level, because such a state – from the thermodynamics point of view – is the most probable one. Instead, in the theory presented here the maximum entropy state is extremely improbable *for any system*, to the extent to which maximum entropy H implies neither equilibrium nor existence at all for the system. At the opposite extreme, also $h = 0$, i.e., a definitively stable equilibrium, is impossible, since it would imply a system consisting of an infinite number of components. See Equation **[20]** ahead).

4.2 - A cardinal theoretical assertion

Equations **[8]**, the number of which is N^2 (the square number of the system's components), together with Equation **[9],** show that all the needed information concerning the study system can be expressed through functions of the interaction flows.

From a practical point of view, this means that the significant amount of information concerning the state of the system can be obtained through any appropriate collection, interpretation and processing of the data that quantify the flows.

However, and this is the fundamental methodological statement, all the information obtained from the theoretical analysis depends strictly on how the system has been identified and described. The simulation theory does not provide any *true picture* of the reality to which the analysis refers, but *only* the logical implications of *a mental representation* of it.

5. A description of the system's evolution

The most important equations provided by the theory are those which enable the analyst to simulate the system's evolution process. This is described by a sequence of "transformation cycles", each cycle developing through discontinuous "transition phases", which are changes in the system's state, each phase being described by a different distribution of the interaction flows.

In every transformation cycle, the condition of the system is expressed by a set of parameters (*state* and *phase parameters*), amongst which *entropy, syntropy* and *stability* are the most significant ones.

Any transformation cycle starts from an "initial phase" (also called "phase zero"), which is determined by any change – however small – in the original flow configuration that modifies the system's *base entropy* defined by Equation **[12].**

The *initial transition phase* of an evolution process is not the ***original state***: this is only the system's first configuration that could be described through a direct survey of the transactions flows, which also provided the first set of observed data.

Instead, the *initial transition phase* is supposed to be the *initial change* in the system's flow distribution *observed* (or *introduced*) after the original one.

Thus, the *initial phase* (also dubbed ***phase zero***) is supposed to be connected with the *original state* through a sequence of *virtual transition phases* that represent the virtual "past story" of *phase zero.*

"Phase zero" is viewed as the *initial phase* of an observed *transformation process*, which is defined by a sequence of *actual transition phases,* each representing a section of the system's simulated future.

It is worth remembering that a sequence of *actual phases* describes *one transformation* ***cycle,*** which may – or may not – be followed by further transformation cycles.

During each cycle, the continuity in the identity of the study system rests on two sets of quantities, dubbed "*structure potentials*" and represented by letters X_k and Y_j , whatever j or k.

Y_j and X_k are non-negative values that remain constant with the system's structure $\{\ m_{jk}\ \}$ during each transformation cycle, and may be considered as obtained from the solution of the two following linear equation systems, respectively:

[15] $\Sigma_j Y_j\, e^{m_{jk}} = 1$; (valid for any k)

[16] $\Sigma_k X_k\, e^{m_{jk}} = 1$; (valid for any j)

where "structure potentials" Y_j and X_k are the unknown terms.

It is also proved that all the 2N "structure potentials" Y_j and X_j verify the following relations**:**

[17] $\Sigma\, Y_j = \Sigma X_j = h$.

The *structure potential* values range between 0 and 1.

Moreover, it is proved true that , in equilibrium states,

[18] $h = X_j / Q_j = Y_j / P_j$ **,** (valid for any j)

which is a useful calculation instrument in simulating a transformation cycle.

Note:
[19] if $E = E_M = H$, then $h = 1$ and $Y_j = X_j = 1/N$, for all j .

The simulation theory does also prove that

[20] $h = e^{E^*} / N^2$.

which explains why no system, in no state, can enjoy permanent stability (see definition **[12]** for E^*, and **[14]** for "stability" S^*), unless the system consists of an *infinite* number of components.

The state of absolute maximum *syntropy*, according to definitions **[10]** and **[11]** above, is also expressed by $S_M = 2\mathbf{Ln}N = H$, while the respective complementary entropy is $E = E^* = 0$. However, from **[13]** and **[14]** we derive that $h = \mathbf{e}^{E^*} / N^2$. Thus, $h = 0$ only if $N = \infty$, in which case the system has zero probability to change its state. Instead, if $N < \infty$, as it is for the *normal consistence* of systems, it is $h > 0$ in all cases, whatever the value for E^*. This means that any identifiable system has always a probability to change its state.

Therefore, logic arguments prove that no system can attain its pertinent maximum syntropy or entropy state. Such maximums must only be considered as asymptotic limits.

Because of Equations **[17]**, ratios Y_j /h and X_j /h (for any j) define formally two probability distributions. According to a reasonable interpretation, such ratios represent the probability for each element of the system to remain in its state of flow *generator* or *flow attractor*, respectively.

During a transformation cycle, the varying flow distribution relevant to each transition phase of the cycle is identified by indexes between parentheses. For example, $P_j(f\text{-}1)$ represents the "output probability" of component j in the phase *(f-1),* which precedes phase *(f)***;** instead, as another example, $Q_k(f{+}1)$ represents the "demand probability" of component k in the phase *(f+1)* immediately subsequent to phase *(f)***;** and so on also for any other varying distribution of flow probabilities proper to the various transition phases of the cycle.

The equations that determine the probability distributions in the *virtual phases* ("the past story of phase zero") are as follows:

[21] $P_j(f\text{-}1) = \Sigma_k P_{jk}(f) = Y_j \Sigma_k Q_k(f)\, e^{m_{jk}}$; (valid for any j)

[22] $P_{jk}(f\text{-}1) = P_j(f\text{-}1) X_k\, e^{m_{jk}}$; (valid for any j and k)

Instead, the equations of the probability distributions in the *actual phases* ("the possible future") are:

[23] $\Sigma_j P_j(f)\, e^{m_{jk}} = Q_k(f\text{-}1) / X_k$; (valid for any k)

[24] $P_{jk}(f) = P_j(f) X_k\, e^{m_{jk}}$; (valid for any j and k)

[25] $\Sigma_k Q_k(f{+}1)\, e^{m_{jk}} = P_j(f) / Y_j$; (valid for any j)

[26] $P_{jk}(f{+}1) = Q_k(f{+}1) Y_j\, e^{m_{jk}}$; (valid for any j and k)

5.1 – Meaning and use of the transition equations

Equations **[21]** and **[22]** simulate the *most probable* way-back (or "past story") towards the system's original configuration recorded through the original survey by which the original unstable equilibrium state of the system has been identified.

In those two sets of equations, the *unknown terms are on the left-hand side*, whereas the polynomial expressions on the right-hand side are known, starting with the data pertaining to the *initial transition phase* (phase "zero"), in which an *alteration* in the *original **base** of the system has either been detected or hypothesised.* Since phase "zero", because of the introduced alterations, is a *transition phase* of a *transformation cycle,* it must accordingly be supposed that it is preceded by a series of *antecedent transition phases.*

It must also be remarked that the 2*N* *structure potentials* X_j and Y_j remain unchanged during the "wayback" to the *original phase*, because such potentials are just the ones that inhere in the *original state* of unstable equilibrium.

Instead, in identifying the alteration occurred, for any reason, in the *initial phase* ("phase zero"), the structure potentials are bound to change at the conclusion of the transformation cycle.

Important to note**:** all the solutions found for Equations **[21]** and **[22],** which regard *virtual transition phases*, are *positive* probability numbers**;** which is compatible with the concept that the "past story" of the system's *initial transition phase* is certain and traceable. The number of the virtual phases depends only on the number of significant decimals used to approximate the probability values.

Equations **[23]-[24]** and **[25]-[26]** regard the most probable series of the system's *future* configurations as described by the *actual transition phases*. Also in those equations the unknown terms are on the left-hand side, whereas the known terms are on the right-hand side. Alike for the *virtual phase equations*, the initial known terms are provided by the configuration of the *initial phase*.

Looking at the mathematical form of all the transition equations, one can observe that the *semi-bases* $\{ O_j \}$ and $\{ D_j \}$, as well as $\{ P_j \}$ and $\{ Q_k \}$, respectively, are closely inter-related**:** in no case it is possible to modify, for instance, semi-base $\{ O_j \}$ (or, correspondingly, $\{ P_j \}$) without implying necessary changes in semi-base $\{ D_j \}$ (or, correspondingly, $\{ Q_k \}$). And vice-versa. To mean that the values of each semi-base may not be changed independently from one another.

From the simulation point of view, this fact entails that alterations affecting the two complementary semi-bases of the *initial transition phase* (with respect to the *original state*) must be mutually compatible, according to equations of type **[23]** or **[26].** Otherwise, it shall be necessary to opt for that of the two semi-bases in which the alterations are considered as more credible and/or significant. The problem regards in particular the *observed or hypothesised alterations* that the analyst introduces in the *original state.*

There is also to allow for possible cases in which the flow distribution and relative probabilities of the original state (which is in an unstable equilibrium) change without involving any changes in the system's base. Such cases regard intrinsic fluctuations in the values of the configuration's elements, which are not sufficient to start a transformation cycle. In other words, the simulation of a transformation cycle can start only if there are permanent modifications that concern also one of the system's semi-bases.

In principle, alterations detected through surveys should always show mutually compatible semi-bases, at least at an acceptable degree of approximation that take into account inevitable uncertainties inherent in the survey and measurement methodology. In this connection, the conduction of appropriate surveys might work as a significant test on the reliability of this simulation theory. To be born in mind, however, tests of the kind should not regard *original states* of precarious equilibrium, but only observed transition phases of transformation cycles.

As an example concerning national economic systems, the planner (or the simulation operator) should opt either for modifying the *output* (production) semi-base $\{ O_j \}$, or the *demand* (input) semi-base $\{ D_j \}$ of the system in its original unstable equilibrium. Any one of the two options implies the mathematical determination of the other one, which therefore goes to determine the respective semi-base of the subsequent *transition phase* (i.e., *phase one*) of the transformation cycle.

At variance with the *virtual phases,* the solutions of the equations from **[23]** to **[26],** which regard the *actual phases,* **do not** necessarily provide *positive probability values.* There is always an actual phase of the cycle for which the equations give (at least one) *negative* solutions. As soon as any *negative solution* appears in the configuration of an *actual transition phase,* the same phase must be considered as the *barrier* that stops the transformation cycle. In practice, it makes no sense accounting for *negative* probabilities, also because of intervening logarithms that would turn the negative values into *imaginary quantities.*

Therefore, such a phase is considered as that of the *system's disappearance* (sort of *decease or collapse of the system)*, unless a transformation of the system occurs (or is established) on the basis of the phase configuration (i.e., the transaction flow distribution) immediately preceding the *decease phase.* This last phase of the system's life is dubbed "agony phase".

The transformation that avoid the system's collapse consists in a change of its structure – i.e., as to the simulation – in the re-calculation of its *structure potentials,* which express the intervened change in the network of expectations that motivate the actions of the system's components and allow the system's "survival". The re-calculation of the changed *structure potentials* X_j and Y_j uses the flow distribution of the "agony phase" as basic data, as if they were the findings of a survey conducted on a **new** *original state* of *unstable equilibrium*. Obviously, the *new equilibrium state* may also be considered as the *original state* of a further transformation cycle**;** and so on.

(It might be interesting to know that – according to a number of experienced applications of this simulation procedure – the "agony phase" is in most cases the one in the cycle that shows the highest degree of organisation (syntropy) achieved by the system, which seems to "claim" at that stage a change in its structure to avoid disintegration.

As a general observation, the simulation appears better balanced and more significative when the alterations shown in the phase zero, with respect to the original state, are of a moderate amount. For the purpose of simulating the probable effects of major alterations, it seems better to introduce these by small instalments, one by one, in subsequent unstable equilibrium states achieved during the simulated evolution of the study system).

6. Summary of the conceptual framework

Simulations are possible only if a complete set of "original" interaction flows is given.

The observed original flow distribution provides the whole set of data that is necessary and sufficient to carry out analyses and to start simulations.

Conventionally, this simulation theory considers any flow distribution obtained from surveys (or other observation operations) as the representation of an *original equilibrium state* of the study system. This original equilibrium state is intrinsically unstable, and the relevant observed configuration shall be taken as a *mean configuration* about which the system fluctuates precariously.

"Intrinsic instability" means, in fact, that reversible fluctuations in the flow distribution within the original configuration are inevitable, and will sooner or later determine an *irreversible permanent* alteration. *Any minimal* ***irreversible*** alteration in the equilibrium flow distribution (or in the relevant probability distribution), *which modifies also the base entropy of the system*, generates a corresponding particular *initial* phase of the system's "transformation cycle",.

All simulations have to start with an initial transition phase, or "phase zero" ($f = 0$).

The *initial transition phase* of a simulation is an alteration – known by hypothesis – in the original equilibrium state**:** the resulting configuration is the *initial phase* of a transformation cycle, which comes from the *original equilibrium state* and will inevitably conclude with either a change in the system's structure or – in an alternative – in the system's disintegration. In fact, the conclusion of any transformation cycle may either consist in a *new* unstable equilibrium state achieved by the system, or in the *collapse* (and disappearance) of the system.

If a new equilibrium state concludes the cycle, another transformation cycle is expected to originate from it.

Any irreversible alteration – however small – in the system's equilibrium, as quantified by the relevant base entropy, is the necessary and sufficient condition to identify only *one particular* transformation cycle amongst an infinite number of possible cycles.

There is an important remark that concerns the different kinds of instability that characterise the *equilibrium states*, which identify both the *original state* and the *conclusive state* of any transformation cycle, with respect to the instability of the transition phases internal to each cycle.

The instability of *equilibrium states* depends on stochastic events, which may or may not happen, and whose nature and intensity are intrinsically unpredictable. Instead, once any irreversible change modifies such equilibrium states, the subsequent phases of every transformation cycle, with relevant degree of instability, are necessary and univocally determined.

From a logical standpoint, any transformation cycle includes the transition phases that have *virtually* preceded the *initial phase* (i.e., *phase zero*), since this one is conventionally considered, by logic consistency, as the consequence of antecedent "virtual" transition phases that have just led to the irreversible alterations showed by the configuration of transition "phase zero".

In simpler terms, any given *initial configuration* or *phase* is only a *transition phase* with its own *given* past history.

That is why, from a theoretical point of view, the state described by the survey (i.e., the *original* configuration) is never seen as the *initial phase* of the evolution process, but only – with respect to the analyst – as the "conventional unstable original equilibrium state" of a *possible* evolution of the study system.

Therefore, in this context, the adjectives "original" and "initial" have quite different meanings. From the *original state* infinite different *initial* transition *phases* of different transformation cycles may alternatively and arbitrarily be identified or defined, upon an infinite number of possible different alterations in the given *original state* (or *original configuration*).

As a conclusion, in this simulation of an evolution process *original state* and *initial phase* do never coincide.

If a system "survives" at the conclusion of a series of transformation cycles, then the series of undergone "transformations" (which characterise the system's overall evolution) consists in a series of changes in the values that express the "intents" (see Equations **[8]**). These "intents" form the "structure" of the system.

During each transformation cycle (which develops between the original equilibrium *state* and the next equilibrium state, if any), the structure of the system is supposed to remain unchanged, while the flow distribution varies phase by phase, up to a critical phase, in which also the structure must change to allow the system to survive. The critical phase (*agony phase*) is that given by the last set of non-negative solutions obtained from Equations **[23]** or **[25]**.

The *leitmotiv* of the simulation logic is as follows**:** solutions { $P_j(f)$ } of *actual phase f* Equations **[23]** (i.e., the transition phases that form the "future section" of the cycle) provide the numerators of the known terms (right hand side) of phase *(f+1)* Equations **[25],** whose solutions { $Q_k(f+1)$ }, in turn, provide the numerators of the known terms of subsequent phase *(f+2)* equations, and so on, until the *agony phase* is attained, which is the actual phase of the cycle that provides the last set of *non-negative solutions.*

Since negative solutions, in terms of both flows and flow probabilities, are with no physical significance, it is conventionally assumed that the represented system *cannot exist further* in the phase that follows the *agony phase*.

Under the assumption that the system will instead survive, the new structure of the system is normally re-calculated in function of the flow distribution inherent in the "agony phase", by use of Equations **[24]**, **[26]** and **[8].**

The assumption of survival, however, is not logically necessary**:** It may or may not be adopted, according to the nature and purposes of the simulation.

Note: At variance with the equations regarding the *actual transition phases,* Equations **[21]** and **[22]**, which relate to virtual transition phases, do always and necessarily provide *non-negative* solutions for *all virtual phases.* The solutions (configurations) obtained from these virtual-phase equations tend to regain the configuration of the *original equilibrium state* of the system, through a reverse-time phase sequence. The number of phases of this virtual sequence is practically determined by the number of significant decimals adopted for the values of the solutions obtained from **[21]** and **[22]** as well as for the values of the *original state*.

The process described by the simulation is that of activities generated by a system of expectations (the *intents*), which tend to be conservative but are necessarily modified, through feedback reactions, by the development of the overall system activity.

Once each cycle is concluded by a transformation that brings the system into a new unstable equilibrium state, a new cycle may start. The cycle that follows assumes the flow distribution resulting from the preceding transformation as a new *original* equilibrium *state* in the system's evolution.

In the same way, further cycles may follow in describing the system's evolution

If, at a certain point in the evolution, the necessary structure transformation does not occur, then the system exits from the area of the conventional reality, and the simulation stops.

It is worth pointing out that a theoretical potential maximum syntropy (level of organisation) inheres in any defined system. The maximum syntropy value coincides with the corresponding "entropy potential" **[10]** of the system**.** In approaching its maximum syntropy, the system's evolution may substantially enter stationary conditions, which actually block any further development.

Further evolution (or involution) is then possible only if the system undergoes a *mutation.* A mutation occurs when the system must be re-defined because of major changes in its features, the nature of which *involves an increase or decrease* in the number of its components (or "sectors", if it is an economic system).

A process of progressive (or regressive) *functional differentiation* within the range of the system's components (or "sectors") may involve a sequence of "mutations".

The main use suggested for the simulation process illustrated here is that proper to a "sensitivity analysis", which is often necessary to test the suitability of planning or political initiatives.

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